

Methods of Applied Mathematics – Honors: New Jersey Institute of Technology Fall Semester

Albert Dorman Honors College
David Reibstein, Dean
Mathematics 450-H01 (senior), 3 credit hours
Professor Bruce Bukiet

Course Description: In this course, students perform and analyze physical experiments in the context of an advanced mathematics course. This capstone course integrates the students' experience with mathematical modeling, mathematical analysis, numerical methods, computation, engineering and communication. In the first semester, students have short modules (2-4 weeks) that include relatively simple experiments and numerical simulations. This prepares students for the second semester, when students work in teams to perform and analyze experiments of greater complexity using more advanced mathematical skills. At the end of the second semester, students present their research results both orally and in writing.

Texts

Textbooks: Haberman, *Mathematical Models: Mechanical Vibrations, Population Dynamics and Traffic Flow*, and Farlow, *Partial Differential Equations for Scientists and Engineers*

Experimental Apparatus: Vernier LabPro – Data acquisition and analysis software, Accelerometer, Photogates, Temperature probe, Masses, Springs, Pendulum, Cycloid track, Power supply, voltmeter, conductive paper and pens

Syllabus

Date	Topic
Unit I: Introduction - Math Modeling, Gravity and Newton's Law of Cooling	
Week 1	Review of Differential Equations, Introduction to Mathematical Modeling and Applied Problems Physical Experiment 1: Newton's Law of Cooling - is the power really 1?
Week 2	Equilibrium and Stability in one dimension (1st order), Newton's Law of Cooling Review vector calculus, Newton's laws, conservative systems
Week 3	Least squares fitting for realistic data Project 1: Mathematical modeling and Newton's Law of Cooling experiment analysis
Unit II: Mechanics I - The Brachistochrone	
Week 4	Calculus of Variations
Week 5	Derivation of the Nonlinear Differential Equation governing the Brachistochrone (Curve for which a ball travels from one point to another in the fastest time under the influence only of gravity), Solution to the Nonlinear Ordinary Differential Equation (Parametric Equations) Physical Experiment 2: Timing a trajectory: the Brachistochrone vs. the line

Week 6	Tautochrone property of the Solution, Analysis for the line and of the cycloid for different height/length ratios Project 2: Calculus of variations, Brachistochrone experiment and analysis of the cycloid
Week 7	Review and Midterm and Going over Midterm
Unit III: Mechanics II - Mass-Spring Systems	
Week 8	Review Midterm, Second order ODEs and harmonic motion, Dimensional Analysis
Week 9	Derivation and solution of undamped and damped single mass-spring systems Physical Experiment 3: Single vertical mass-spring setup
Week 10	Phase plane analysis, Double mass-spring system, Non-linear oscillations and the Pendulum Project 3: Measuring the spring constant, frequency and evaluating linearity of a spring and other mass-spring analysis
Week 11	Linear Stability and Linearization (higher order), Energy Conservation and Energy Curves, Numerical Methods for ODEs Physical Experiment 4: Double mass-spring and its frequencies Project 4: Double mass-spring and its frequencies; how initial conditions influence the dynamics of the double mass-spring; nonlinear springs
Week 12	Phase curves for the damped pendulum, The Spring Pendulum Project + Physical Experiment 5: Timing the pendulum, analysis of the nonlinear pendulum and linearized pendulum equations
Unit IV: Electrostatics and Incompressible Fluids	
Week 13	Derivation of Laplace equation for potential flow, Electrostatic potential, Properties of the Laplace equation, Elliptic PDEs
Week 14	Separation of Variables, Solutions in Rectangular and Cylindrically symmetric regions
Week 15	Finite difference methods, Review Physical Experiment 6: Electrostatic Field Mapper experiment Project 6: Analytic and Experimental Solution of Laplace's equation for electrostatics problems (equipotential and flux lines)

Grading Policy:

The final grade in this course will be determined as follows:

Homework/Projects: 66%

Midterm and Final Exams: 34%

**Methods of Applied Mathematics – Honors: New Jersey Institute of Technology
Spring Semester**

**Albert Dorman Honors College
David Reibstein, Dean**

Mathematics 451-H02 (senior), 3 credit hours
Professor Roy Goodman

Course Description: In the spring semester, we will learn more advanced methods from classical mechanics and use them to study problems that have attracted more recent interest: dynamical bias in coin tosses, as shown by Diaconis et al., chaos in the double pendulum, and the dynamics of simple walking toys.

Texts

H.C. Corben and Philip Stehle, *Classical Mechanics XYZ* and expository articles: Keller, "The Probability of Heads," Amer. Math. Mo., (93) 1986; Diaconis, Holmes, Montgomery, "Dynamical Bias in the Coin Toss," preprint, 2004, Halir & Flusser, "Numerically stable direct least squares fitting of ellipses," Proc 6th Intl. Conf. in Central Eur. On Computer Graphics, 1998, papers by McGeer, and the Ruina lab on walking toys

Experimental apparatus: Matlab image processing toolbox, digital camera, high-speed video camera & software, gyroscopes, coins, plates, and pendula

Syllabus

Date	Topic
Unit I: Rigid body mechanics	
Week 1	Course overview, introduction to Matlab's image processing toolbox, Keller's "no dynamics" coin-toss model Project 1: Use Matlab image processing software to track object in video
Week 2	Review vector calculus, Newton's laws, conservative systems
Week 3	The gravitational potential, review of variational methods, Hamilton's principal and derivation of equations of motion as Euler-Lagrange equations Project 2: Extend project 1 to calculate gravitational acceleration from a video of a bouncing ball
Week 4	Conservation laws and symmetry, rigid rotations in two dimensions, moments of inertia, parallel axis theorem
Week 5	Rigid rotation in 3D, parallel axis theorem, body frame & fixed frame, rotational kinetic energy & the inertia tensor, angular momentum. More image processing, least squares fitting & specialized methods for fitting ellipses Project 3a: Feynman's plate experiment part I: shoot and analyze video of thrown dinner plate, detect edges and fit to ellipses
Week 6	Euler's equations, the rotator, the symmetric free top, Feynman's plate experiment, geometry of three-dimensional reconstruction of plate from image Project 3b: Feynman plate II: reconstruct plate positions, verify analytic prediction
Week 7	The asymmetric free top, stability of motion about axes, the Poincot sphere Project 3c: Experimental verification of stability and instability

Week 8	Moving between fixed and body frame, the body cone & space cone, the Diaconis et al. result Begin big project A: Shoot and analyze several high-speed videos of coin tosses to verify the Diaconis result and get a probability distribution of biases
Week 9	Parallel axis theorem for inertia tensors, the “heavy top” (gyroscope) Project 4: The gyroscope
Unit II: Pendulums and nonlinear oscillators	
Week 10	Forced damped linear and nonlinear oscillators, Poincare maps, chaos Project 5: Forced damped linear and nonlinear oscillators
Week 11	Stabilization of the inverted pendulum by rapid oscillation of support (with demonstration!)
Week 12	The double pendulum, Lyapunov exponents Project 6: Numerical and experimental demonstration of chaos using Lyapunov exponents
Week 13 - 14	Modeling and experiments with a simple walking toy, reference to Ruina lab
Week 15	Practice project presentations

Grading Policy: The final grade in this course will be determined as follows:

Homework exercises: 25%

Projects and Presentations: 75%

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